Week 14:
Electromagnetic waves

Structure of the lecture

1. Waves
    1. What is a wave?
    2. 1D wave equation
    3. Sinusoidal plane waves
2. Electromagnetic waves
    1. Recall: Maxwell’s equation
    2. How was EM wave discovered? Hertz experiment
    3. How to produce EM waves?
    4. Maxwell’s equation in vacuum
    5. Electromagnetic waves in vacuum
    6. Electromagnetic spectrum
    7. Summary of the lecture
3. Some solved problems

What is a wave?

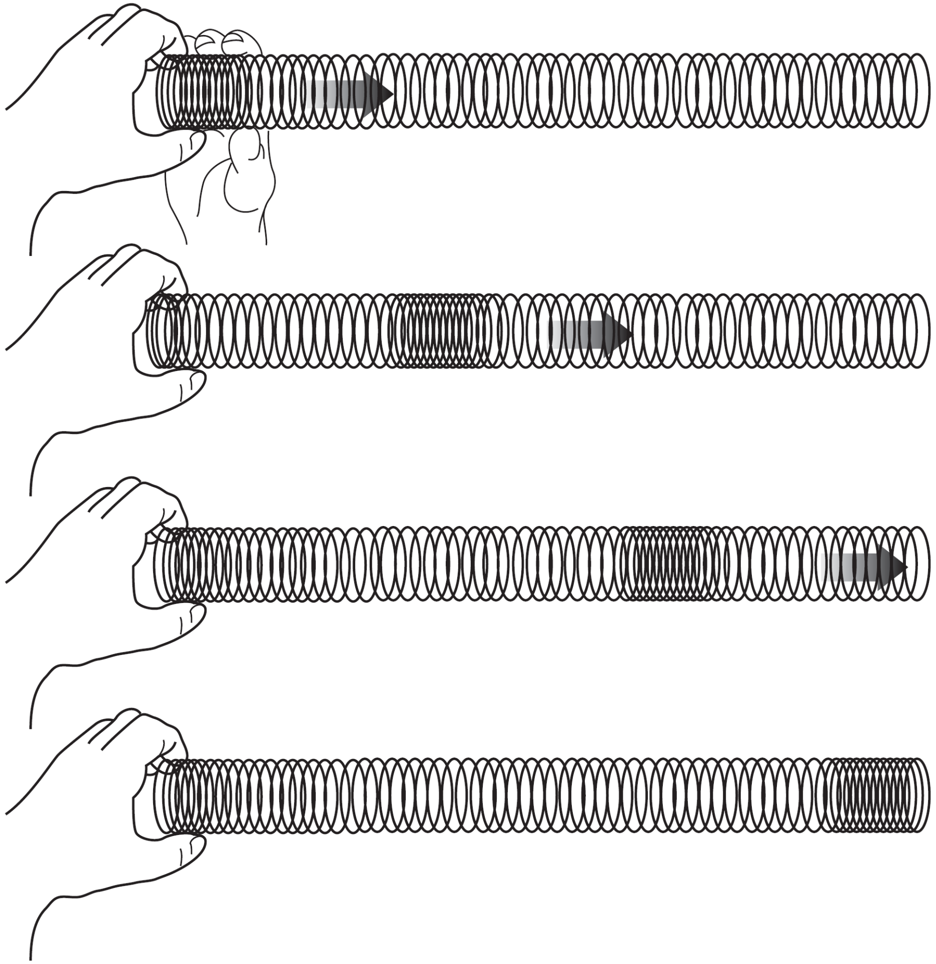
Definition: A wave is a disturbance, $\psi(x,t)$, of a continuous medium that propagates with a fixed shape at constant velocity v.

We distinguish two types of waves:

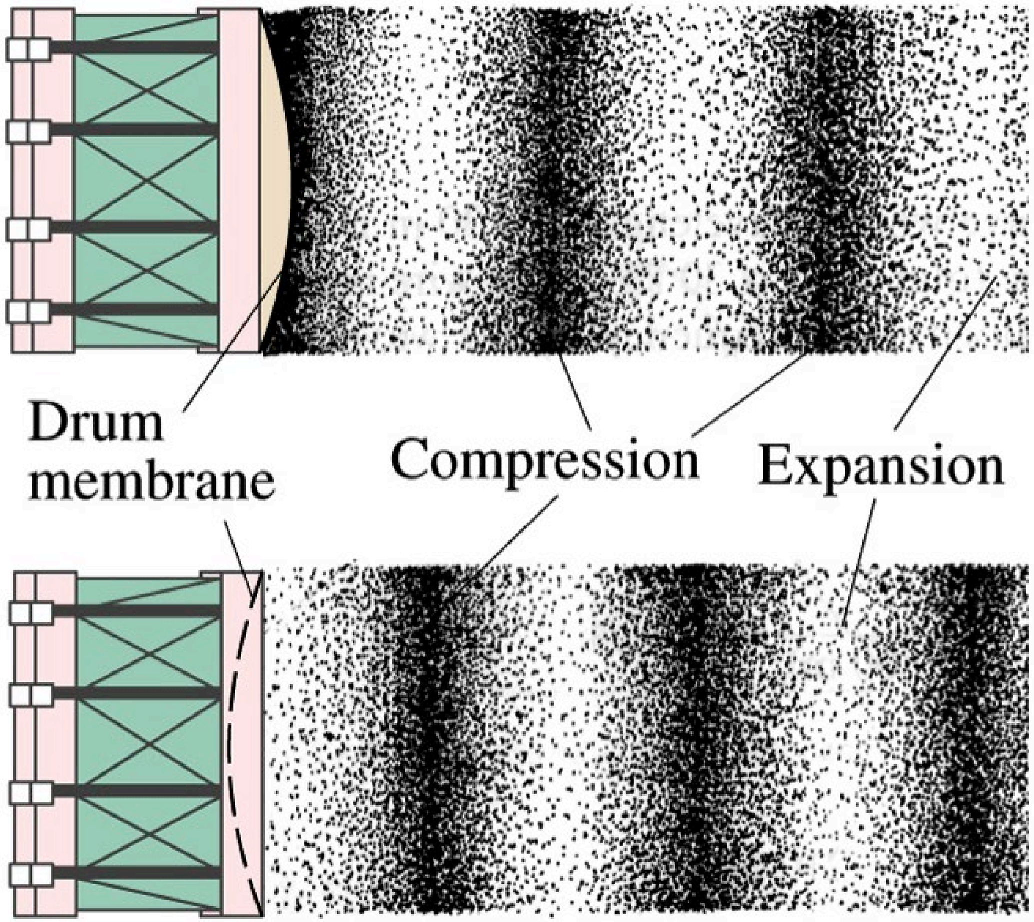
- Longitudinal: **Parallel oscillations** to the propagation direction.
- Transverse: **Perpendicular oscillation** to the propagation direction.

Longitudinal waves

Longitudinal wave in a spring

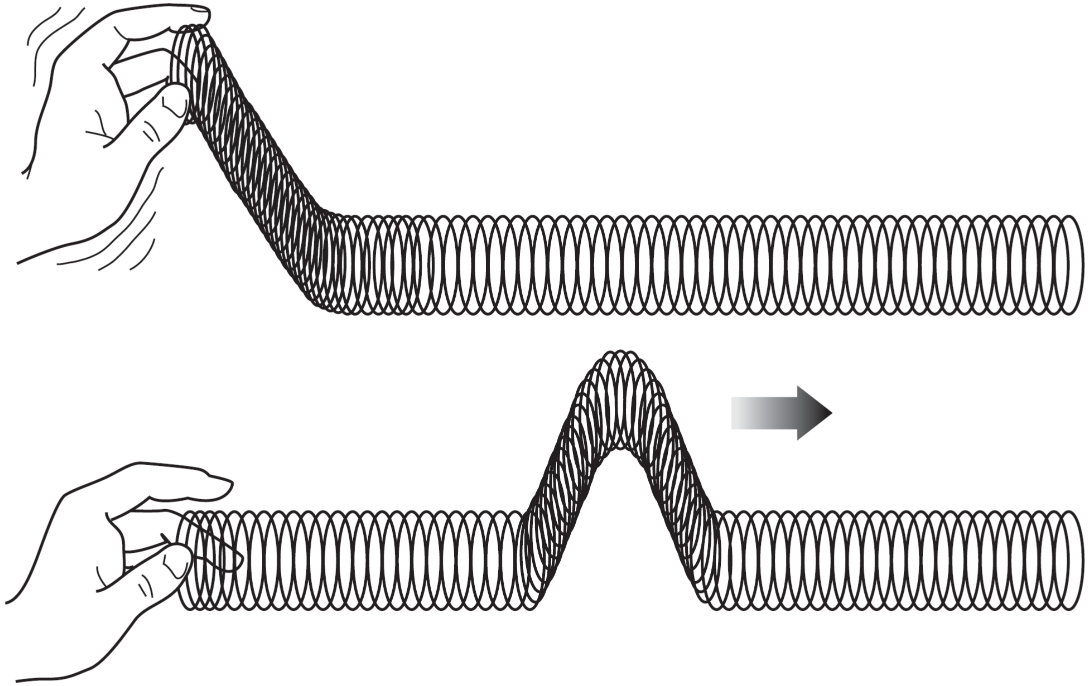


Longitudinal wave in a gas

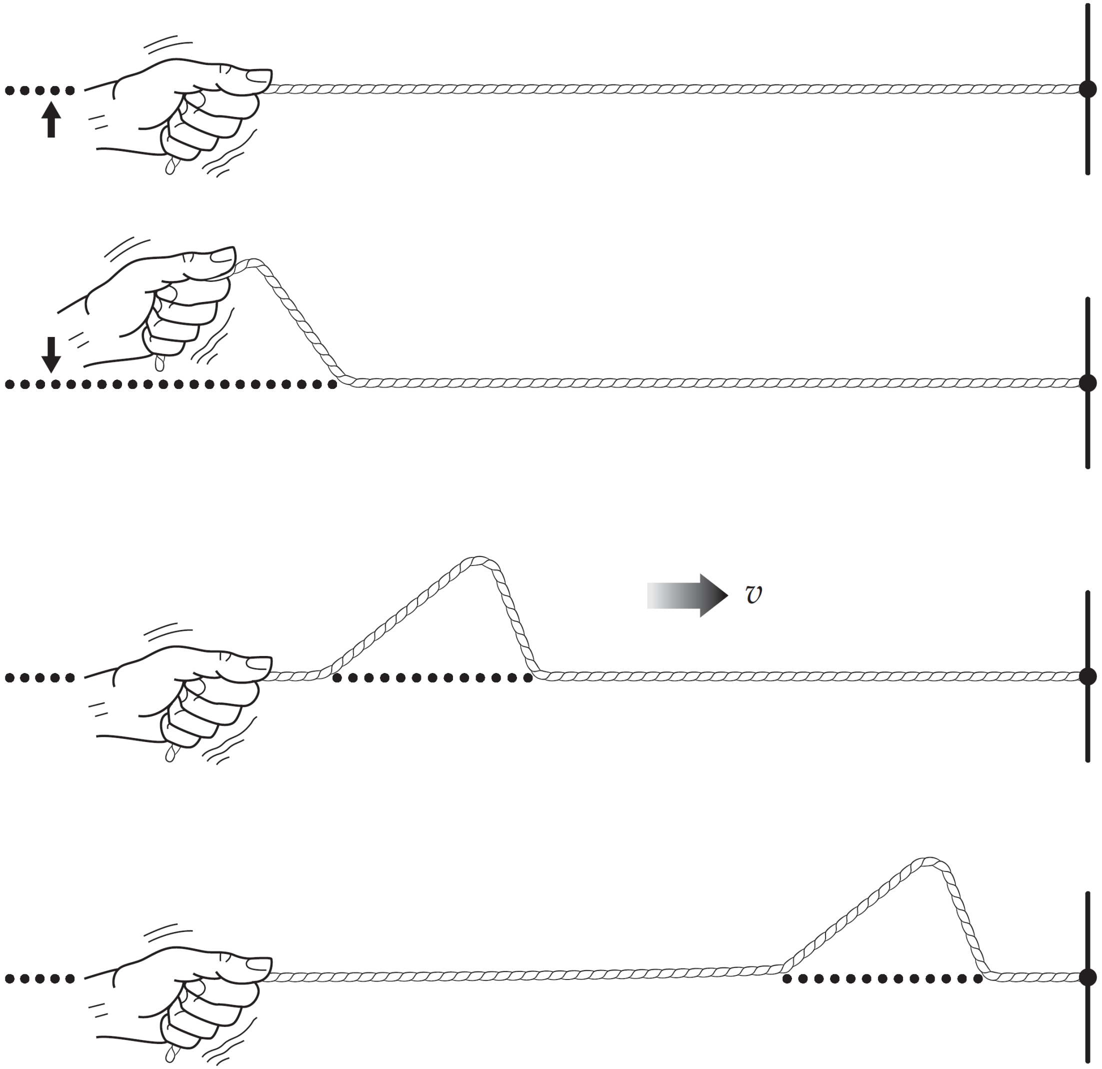


Transverse waves

transverse wave in a spring

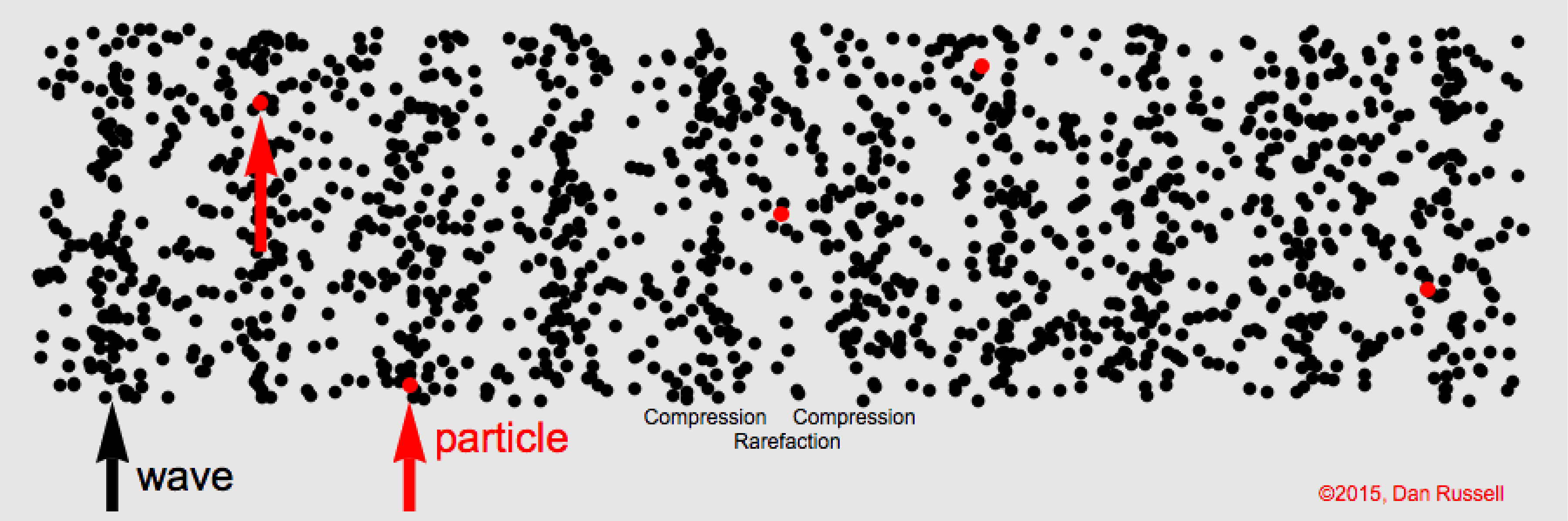


transverse wave in a rope

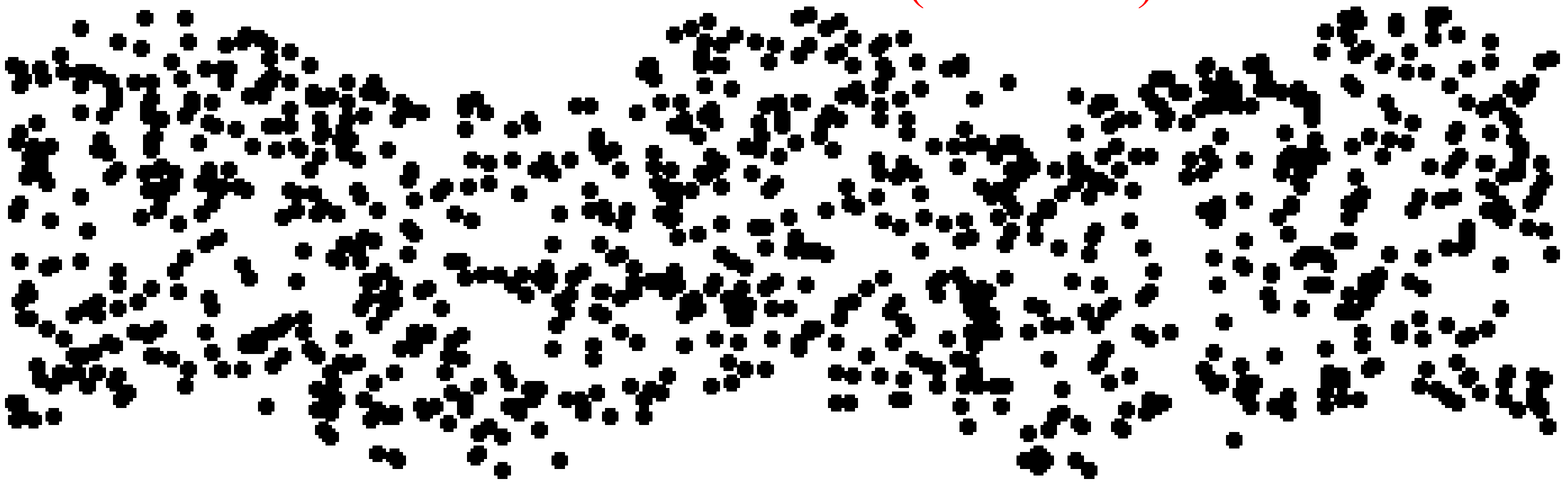


Acoustic waves

Longitudinal wave (animation)



Transverse wave (animation)



What is a wave?

Definition: A wave is a disturbance, $\psi(x,t)$, of a continuous medium that propagates with a fixed shape at constant velocity v.

We distinguish two types of waves:

- Longitudinal: **Parallel oscillations** to the propagation direction.
- Transverse: **Perpendicular oscillation** to the propagation direction.

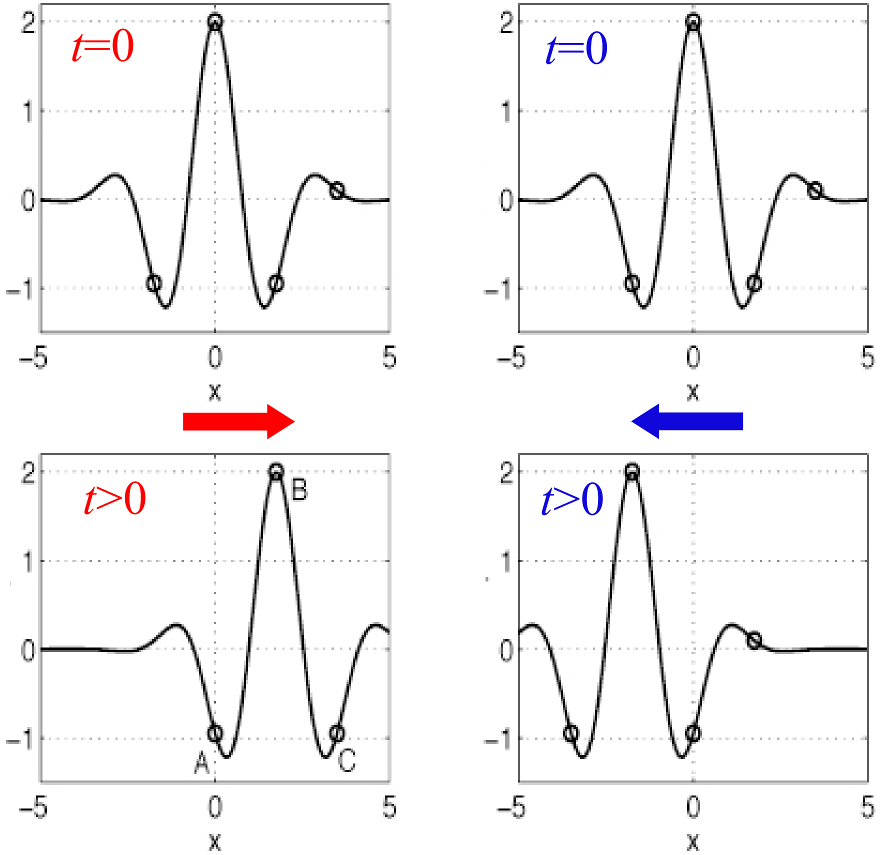
$$\frac{\partial^2\psi}{\partial t^2} = v^2 \frac{\partial^2\psi}{\partial x^2}$$

Equation of d'Alembert
one-dimensional or differential equation of the one-dimensional wave motion

The general solution of the d'Alembert equation is:

$$\psi(x,t) = f(x - vt) + f(x + vt)$$

The d'Alembert wave equation is an example of a linear differential equation, which means that if ψ1 (x, t) and ψ2 (x, t) are solutions to the wave equation, then ψ1 (x, t) ±ψ2 (x, t) is also a solution.
The implication is that waves solution of the d'Alembert equation (and so also electromagnetic waves, as we will see) obey the superposition principle.



traveling wave   *back wave*

One-dimensional differential wave equation

Differential equation of wave motion one-dimensional:

$$\frac{\partial^2\psi}{\partial t^2} = v^2\frac{\partial^2\psi}{\partial x^2} \quad \text{or} \quad \frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

General solution:

$$\psi(x,t) = f(x-vt) + f(x+vt)$$

CURIOSITY

Démonstration:

Let's put: $u = x \pm vt$

$\Rightarrow$

$$\frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial u}\frac{\partial u}{\partial x} = \frac{\partial\psi}{\partial u} \qquad \frac{\partial\psi}{\partial t} = \frac{\partial\psi}{\partial u}\frac{\partial u}{\partial t} = \pm v\frac{\partial\psi}{\partial u}$$

Taking the second derivatives we obtain«

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial u}\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial u}\left(\frac{\partial\psi}{\partial x}\right)\frac{\partial u}{\partial x} = \frac{\partial^2\psi}{\partial u^2} \qquad \frac{\partial^2\psi}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial u}\frac{\partial u}{\partial t}\right) = \frac{\partial}{\partial u}\left(\frac{\partial\psi}{\partial t}\right)\frac{\partial u}{\partial t} = v^2\frac{\partial^2\psi}{\partial u^2}$$

$\Rightarrow$

$$\frac{\partial^2\psi}{\partial t^2} = v^2\frac{\partial^2\psi}{\partial x^2}$$

The three-dimensional wave differential equation

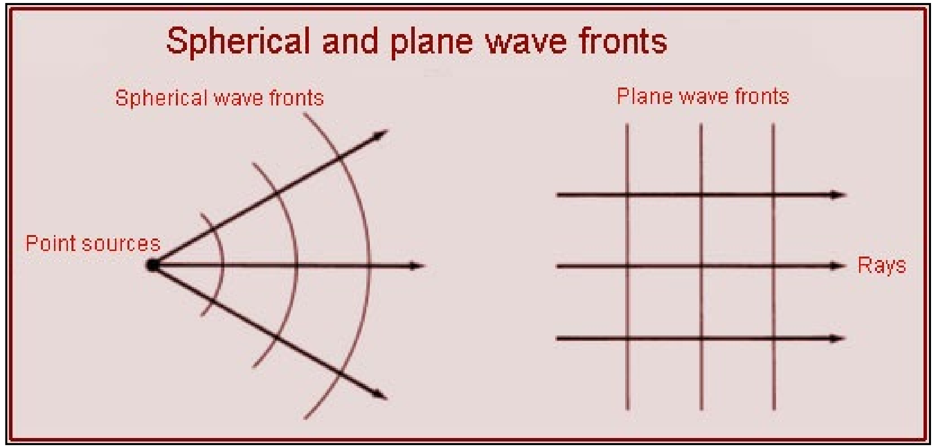
1D
$$\frac{\partial^2\psi}{\partial t^2} = v^2 \frac{\partial^2\psi}{\partial x^2}$$

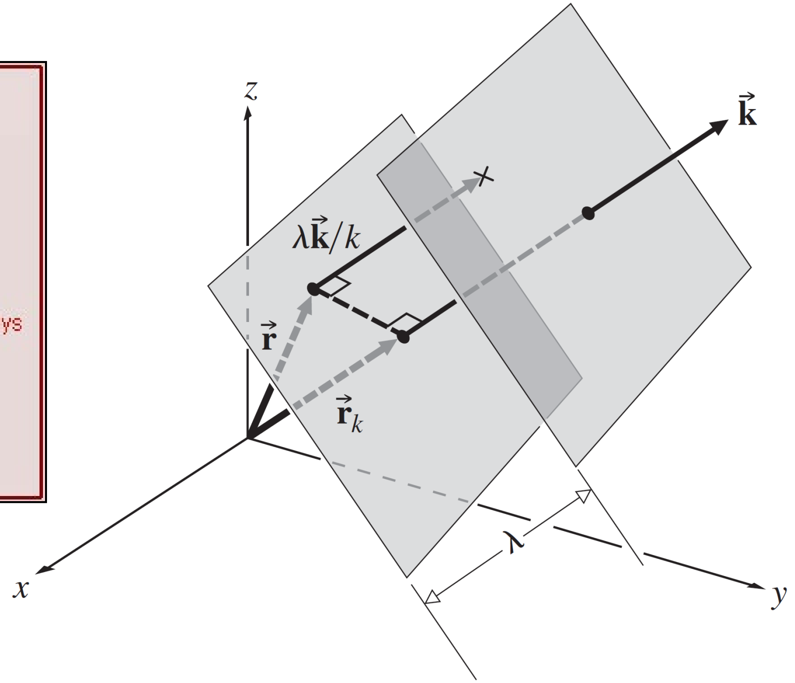
Equation of d'Alembert
one-dimensional or differential equation of the wave motion one-dimensional

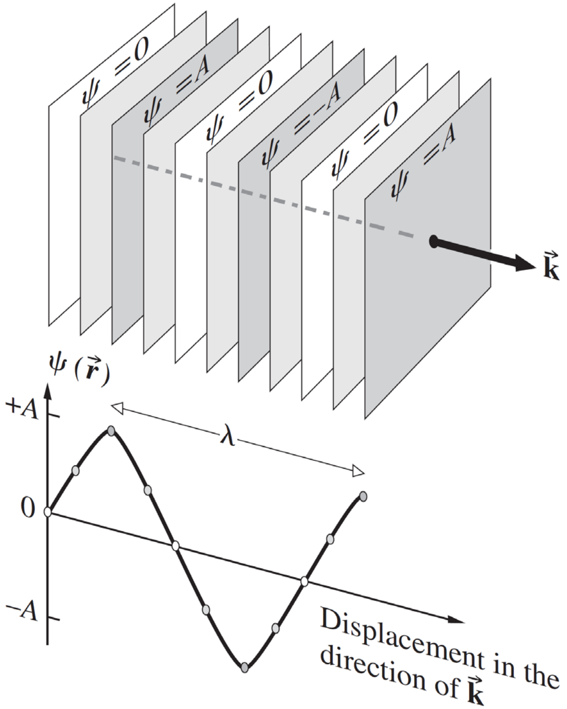
3D
$$\frac{\partial^2\psi}{\partial t^2} = v^2 \left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) = v^2\nabla^2\psi$$

Equation of d'Alembert
three-dimensional or differential equation of the wave motion three-dimensional

Plane waves with arbitrary propagation direction







Plane sine wave propagating along the axis $\hat{\mathbf{k}}$: $\psi(\mathbf{r},t) = A\sin(\mathbf{k}\cdot\mathbf{r} \pm \omega t)$

wave−vector $\mathbf{k}$: $\mathbf{k} = k\hat{\mathbf{k}}$ $k = \sqrt{k_x^2 + k_y^2 + k_z^2} = \dfrac{2\pi}{\lambda} = \dfrac{\omega}{v}$

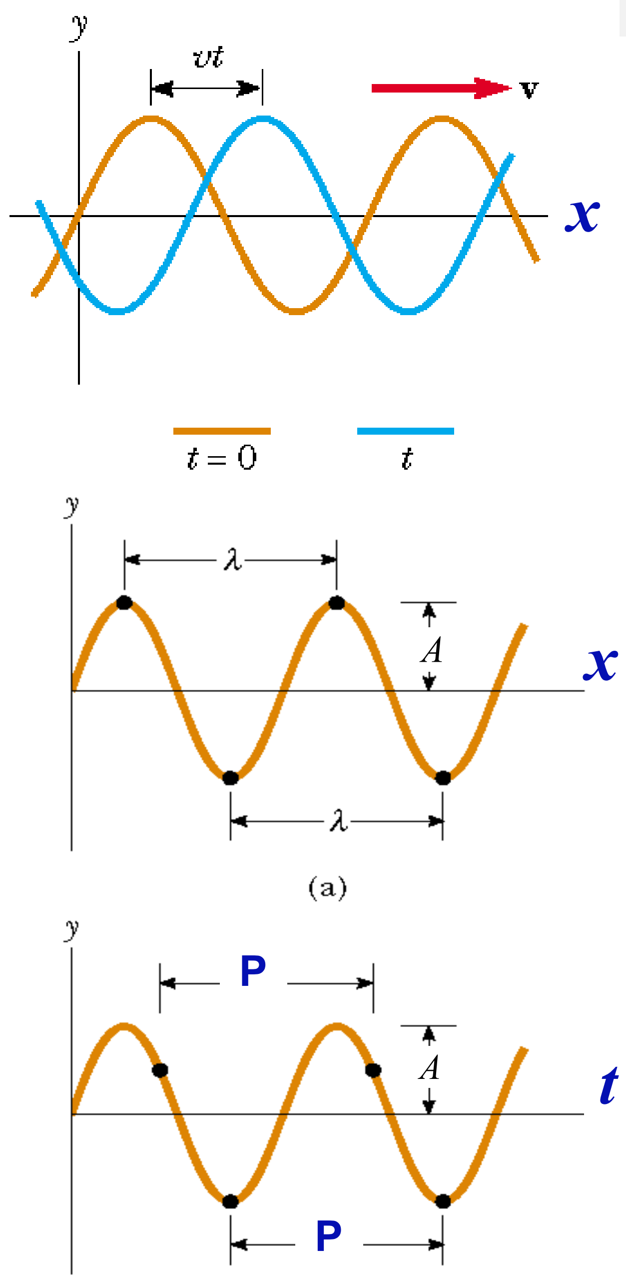
⇒

$\psi(\mathbf{r},t) = A\sin(\mathbf{k}\cdot\mathbf{r} \pm \omega t) = A\sin(k_x x + k_y y + k_z z \pm \omega t)$

(complex form: $\psi(\mathbf{r},t) = Ae^{i(\mathbf{k}\cdot\mathbf{r}\pm\omega t)}$)

Plane waves are a special case of waves where a physical quantity, such as phase, is constant over a plane that is perpendicular to the direction of wave travel.

Sinusoidal plane wave with propagation along x



Sine wave (equivalent shapes):

$$\psi(x,t) = A\sin[k(x \pm vt)] =$$
$$= A\sin(kx \pm \frac{2\pi}{\lambda}vt) =$$
$$= A\sin(\frac{2\pi}{\lambda}(x \pm vt)) =$$
$$= A\sin(2\pi(\frac{x}{\lambda} \pm \frac{t}{P})) =$$
$$= A\sin(kx \pm \omega t)$$ (more common form)

λ	Wavelength[m]
$k = \frac{2\pi}{\lambda}$	Wave number [m^{-1}]
$v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{P} = \lambda f = \frac{\omega}{k}$	phase velocity[m/s]
$f = \frac{v}{\lambda} = \frac{\omega}{2\pi}$	Frequency [Hz]
$P = \frac{1}{f}$	Period [s]

Electromagnetic Waves

Maxwell's Equations (integral form)

We now present **four equations** that are regarded as the basis of all electrical and magnetic phenomena. These equations, developed by Maxwell, are as fundamental to electromagnetic phenomena as Newton's laws are to mechanical phenomena.
The theory that Maxwell developed turned out to also be in agreement with the special theory of relativity, as Einstein showed in 1905.

Maxwell's equations represent the laws of electricity and magnetism that we have already discussed, but they have additional important consequences. For simplicity, we present Maxwell's equations as applied to free space, that is, in the absence of any dielectric or magnetic material. The four equations are:

(1) $$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}$$ ◀ Gauss's law

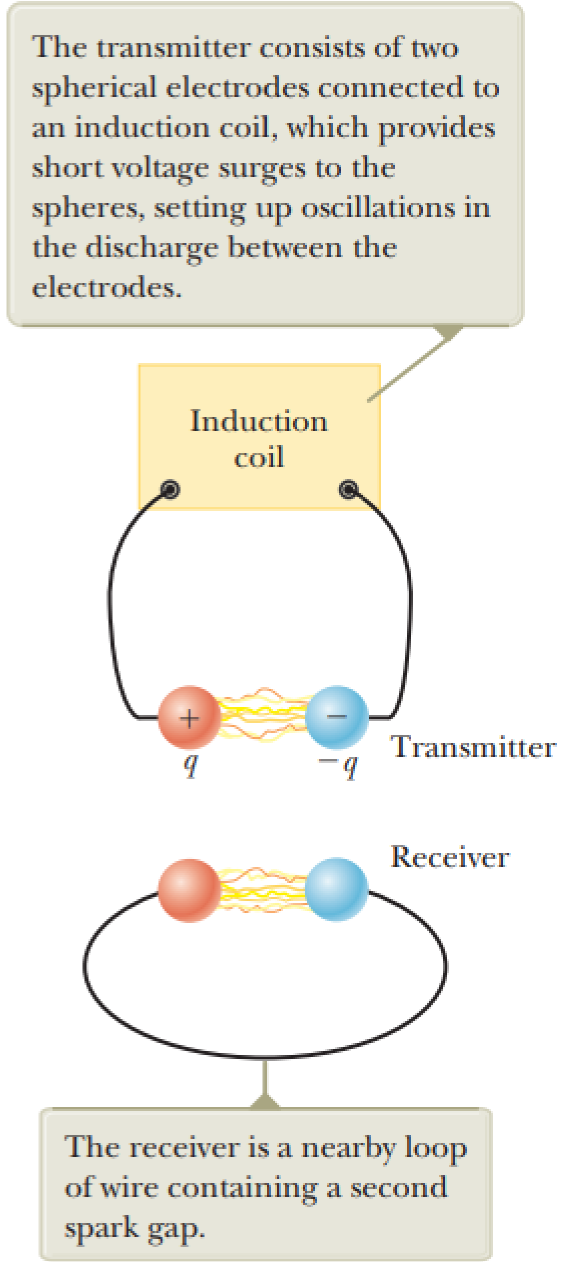
(2) $$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$ ◀ Gauss's law in magnetism

(3) $$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$ ◀ Faraday's law

(4) $$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$ ◀ Ampère–Maxwell law

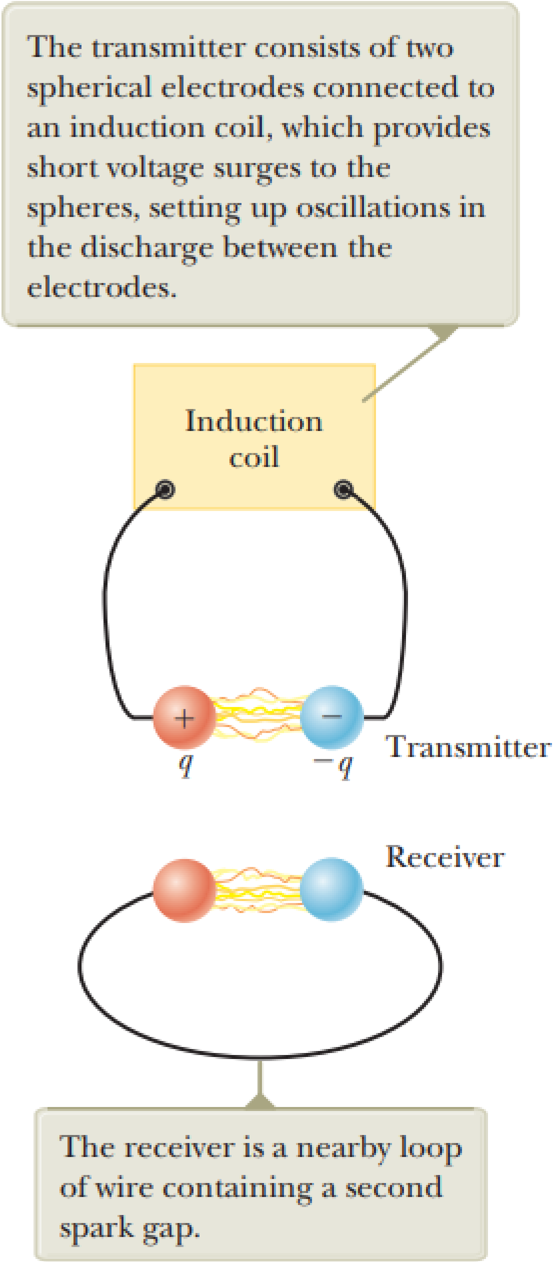
Notice the symmetry of Maxwell's equations. Equations (1) and (2) are symmetric, apart from the absence of the term for magnetic monopoles in Equation (2). Furthermore, Equations (3) and (4) are symmetric in that the line integrals of **E** and **B** around a closed path are related to the rate of change of magnetic flux and electric flux, respectively.

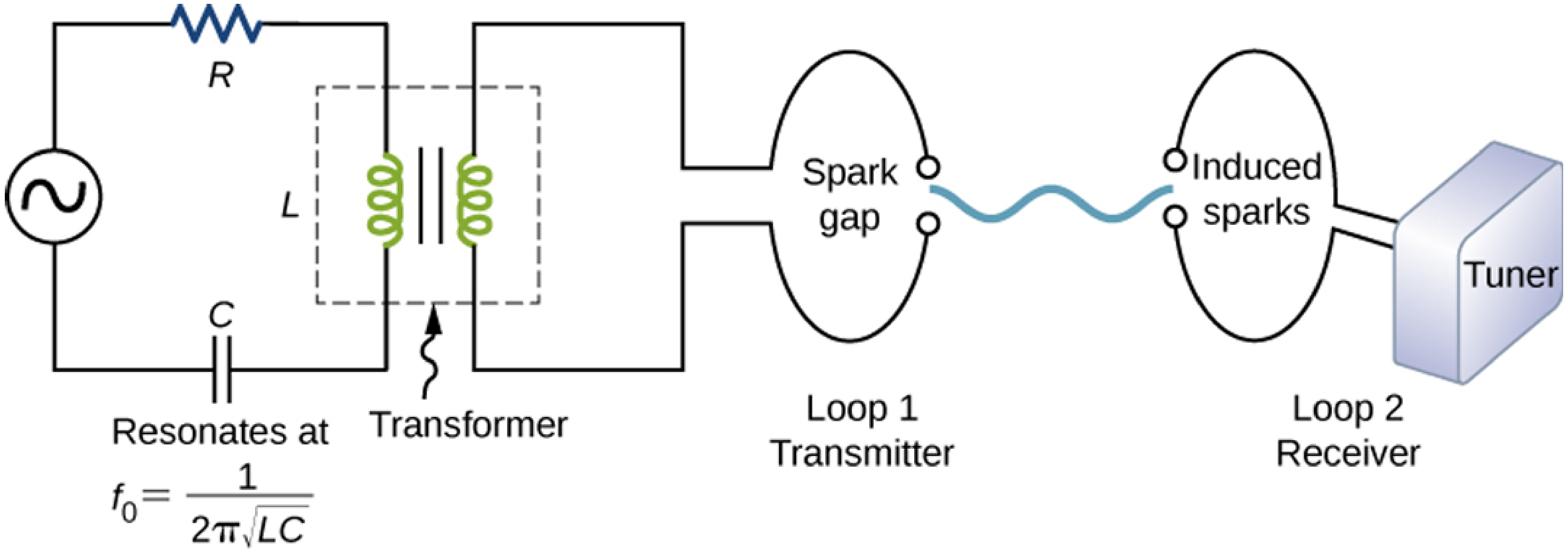
Hertz’s Discoveries



- Hertz performed experiments that verified Maxwell’s prediction. The experimental apparatus Hertz used to generate and detect electromagnetic waves is shown schematically in Figure.
- An induction coil is connected to **a transmitter** made up of two spherical electrodes separated by a narrow gap. The coil provides short voltage surges to the electrodes, making one positive and the other negative. A spark is generated between the spheres when the electric field near either electrode surpasses the dielectric strength for air (3×10^6 V/m).
- From an electric-circuit viewpoint, this experimental apparatus is equivalent to an LC circuit in which the inductance is that of the coil and the capacitance is due to the spherical electrodes.

Hertz's Discoveries





The apparatus used by Hertz in 1887 to generate and detect electromagnetic waves.

Because L and C are small in Hertz's apparatus, the frequency of oscillation is high, on the order of 100 MHz. Electromagnetic waves are radiated at this frequency as a result of the oscillation of free charges in the transmitter circuit.

Hertz was able to detect these waves using a single loop of wire with its own spark gap (the receiver). Such **a receiver** loop, placed several meters from the transmitter, has its own effective inductance, capacitance, and natural frequency of oscillation.
In Hertz's experiment, sparks were induced across the gap of the receiving electrodes when the receiver's frequency was adjusted to match that of the transmitter.

Production of Electromagnetic Waves

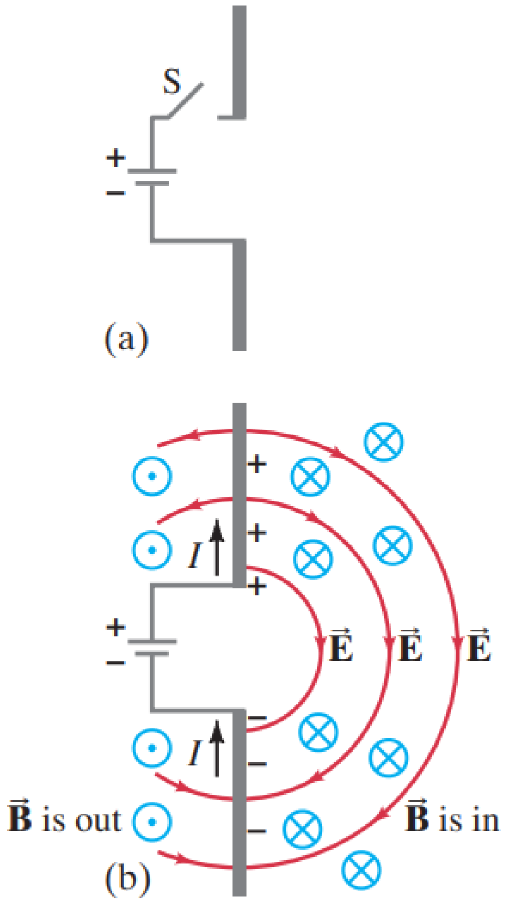


FIGURE 6 Fields produced by charge flowing into conductors. It takes time for the $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields to travel outward to distant points. The fields are shown to the right of the antenna, but they move out in all directions, symmetrically about the (vertical) antenna.

A magnetic field will be produced in empty space if there is a changing electric field.
A changing magnetic field produces an electric field that is itself changing.
This changing electric field will, in turn, produce a magnetic field, which will be changing, and so it too will produce a changing electric field; and so on.

Maxwell found that the net result of these interacting changing fields was a wave of electric and magnetic fields that can propagate (travel) through space!

- Consider two conducting rods that will serve as an “antenna”. Suppose these two rods are connected by a switch to the opposite terminals of a battery. When the switch is closed, the upper rod quickly becomes positively charged and the lower one negatively charged.

- Electric field lines are formed as indicated in Figure. While the charges are flowing, a current exists whose direction is indicated by the black arrows. A magnetic field is therefore produced near the antenna. The magnetic field lines encircle the rod-like antenna and therefore points into the page on the right and out of the page on the left.

- In the static case, the fields extend outward indefinitely far.

However, when the switch is closed, the fields quickly appear nearby, but it takes time for them to reach distant points. Both electric and magnetic fields store energy, and this energy cannot be transferred to distant points at infinite speed.

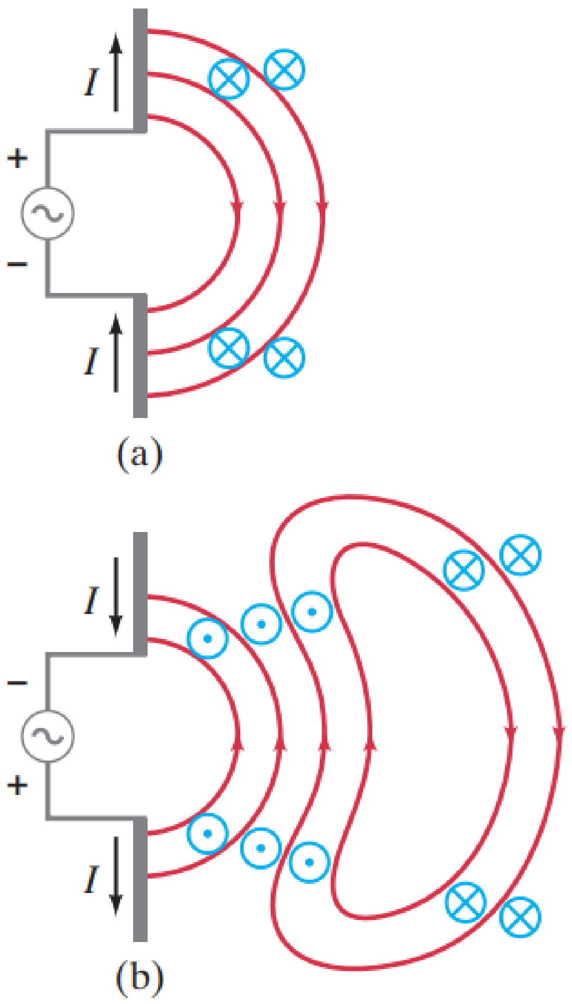


FIGURE 7 Sequence showing electric and magnetic fields that spread outward from oscillating charges on two conductors (the antenna) connected to an ac source (see the text).

Now we look at a different situation, where our **antenna is connected to an ac generator.**

In Fig. a, the connection has just been realized. Charge starts building up and fields form.

The + and - signs in Fig. a indicate the net charge on each rod at a given instant.

The black arrows indicate the direction of the current *I*. The electric field is represented by the red lines in the plane of the page; and the magnetic field, according to the right-hand rule, is into or out of the page, in blue

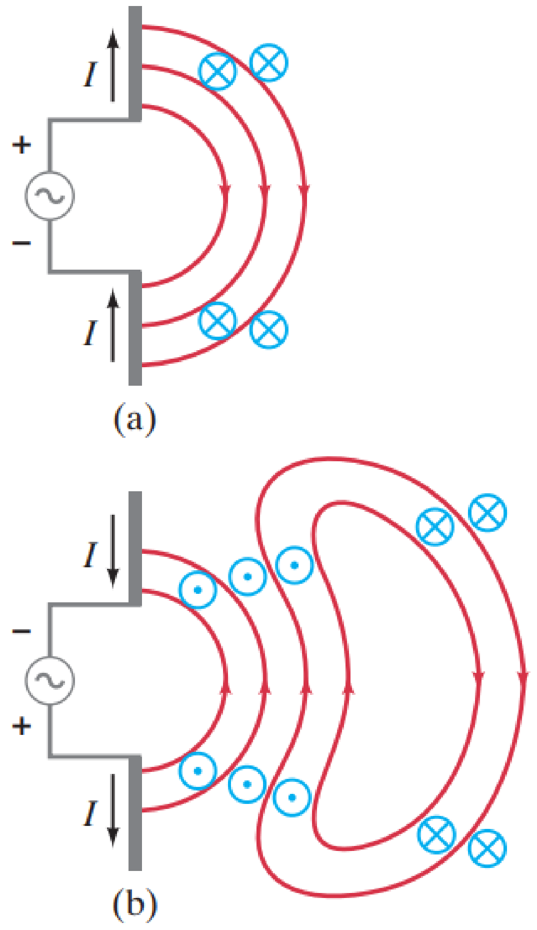
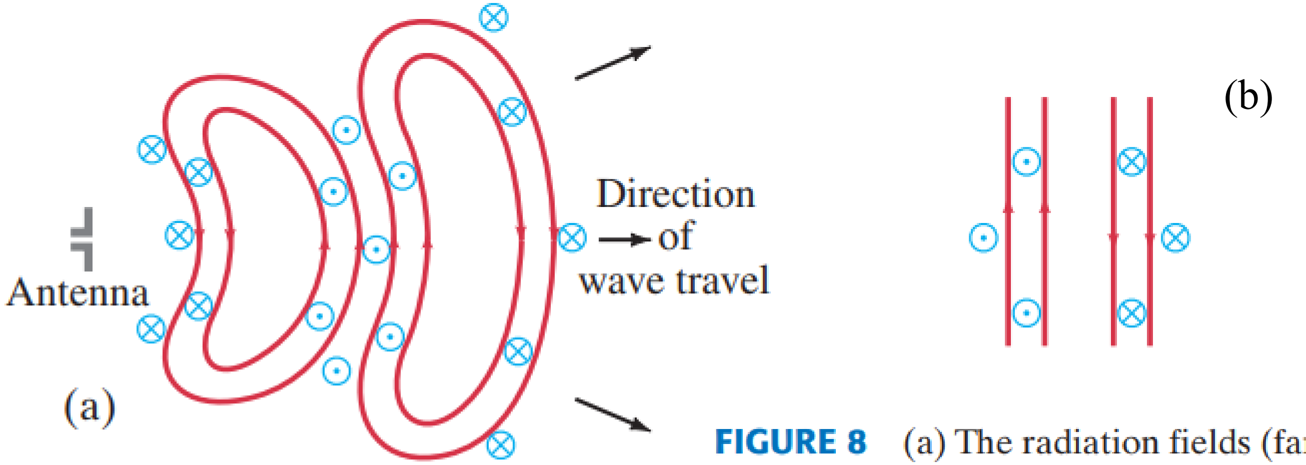


FIGURE 7 Sequence showing electric and magnetic fields that spread outward from oscillating charges on two conductors (the antenna) connected to an ac source (see the text).

In Fig. b, the voltage of the ac generator has reversed in direction; the current is reversed, and the new magnetic field is in the opposite direction. Because the new fields have changed direction, the **old lines fold back** to connect up to some of the new lines **and form closed loops,** as shown.

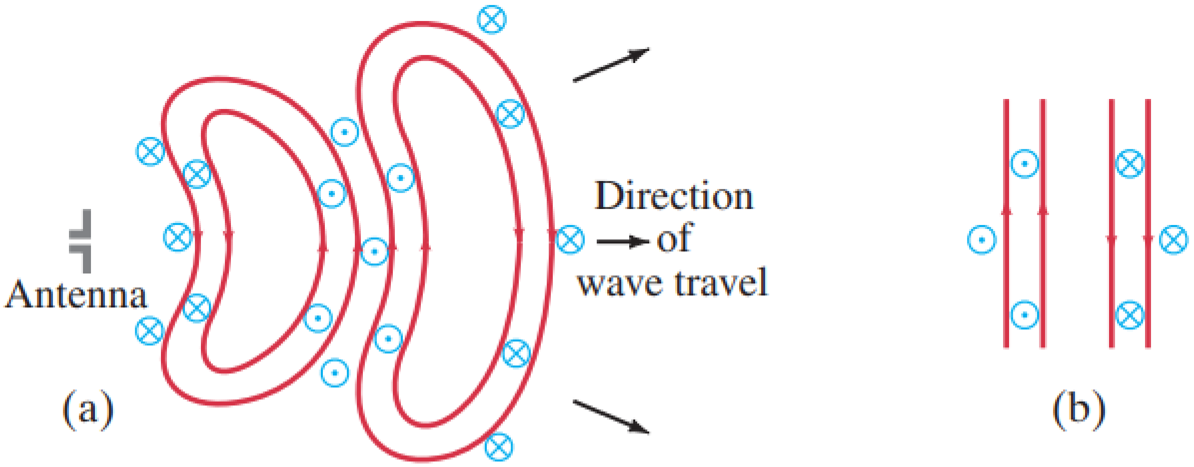
The old fields, however, don't suddenly disappear; they are on their way to distant points. Indeed, because a changing magnetic field produces an electric field, and a changing electric field produces a magnetic field, **this combination of changing electric and magnetic fields moving outward is self-supporting**, no longer depending on the antenna charges.



SELF SUPPORTING WAVES

FIGURE 8 (a) The radiation fields (far from the antenna) produced by a sinusoidal signal on the antenna. The red closed loops represent electric field lines. The magnetic field lines, perpendicular to the page and represented by blue ⊗ and ⊙, also form closed loops. (b) Very far from the antenna the wave fronts (field lines) are essentially flat over a fairly large area, and are referred to as *plane waves*.

The fields *not far* from the antenna, referred to as the ***near field***, become quite complicated.
We are instead mainly interested in the fields *far from the antenna*, which we refer to as the ***radiation field, or far field***. The electric field lines form loops, as shown in Fig., and continue moving outward.
The magnetic field lines also form closed loops but are not shown since they are perpendicular to the page.



Several things about the radiation field can be noted from this Fig..

(1) the electric and magnetic fields at any point are perpendicular to each other, and to the direction of wave travel.

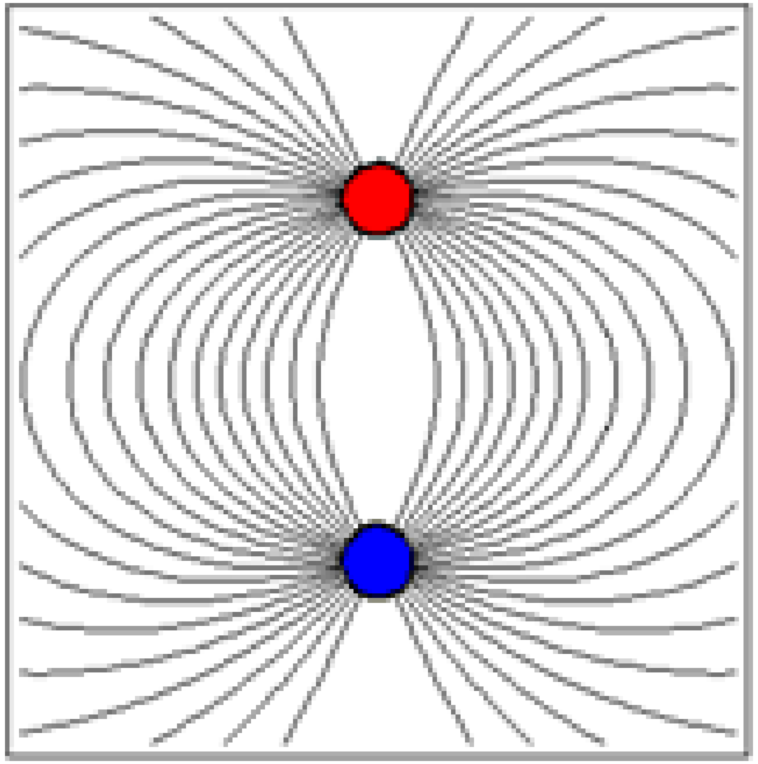
(2) we can see that the fields alternate in direction (**B** is into the page at some points and out of the page at others; **E** points up at some points and down at others). Thus, the field strengths vary from a maximum in one direction, to zero, to a maximum in the other direction.

(3) the electric and magnetic fields are "in phase": that is, they each are zero at the same points and reach their maxima at the same points in space.

(4) very far from the antenna the field lines are quite flat over a reasonably large area, and the waves are referred to as **plane waves**.

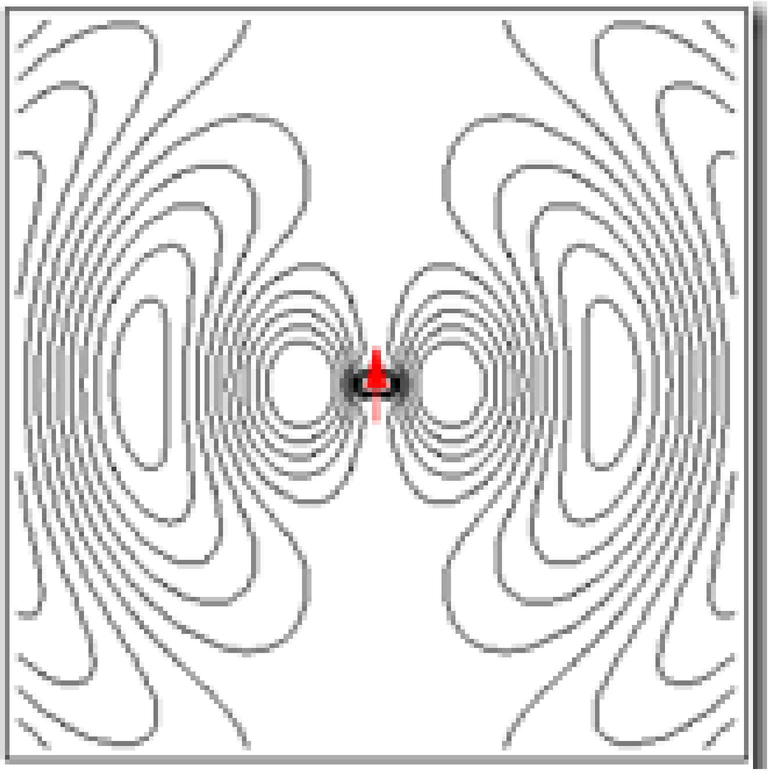
Static Electric Dipole

(generates a static electric field)

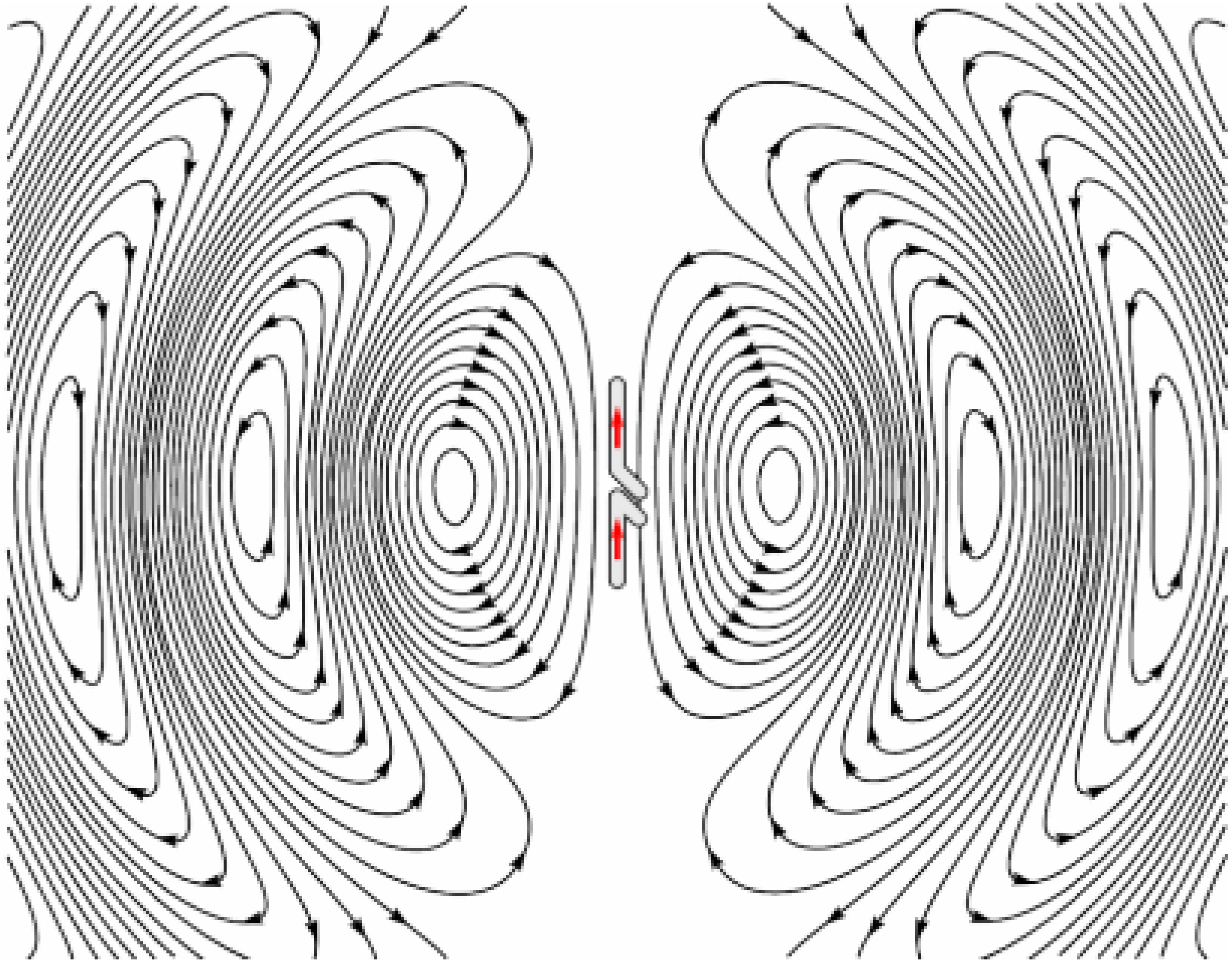


Oscillating Electric Dipole

(generates an oscillating electric field and an oscillating magnetic field)

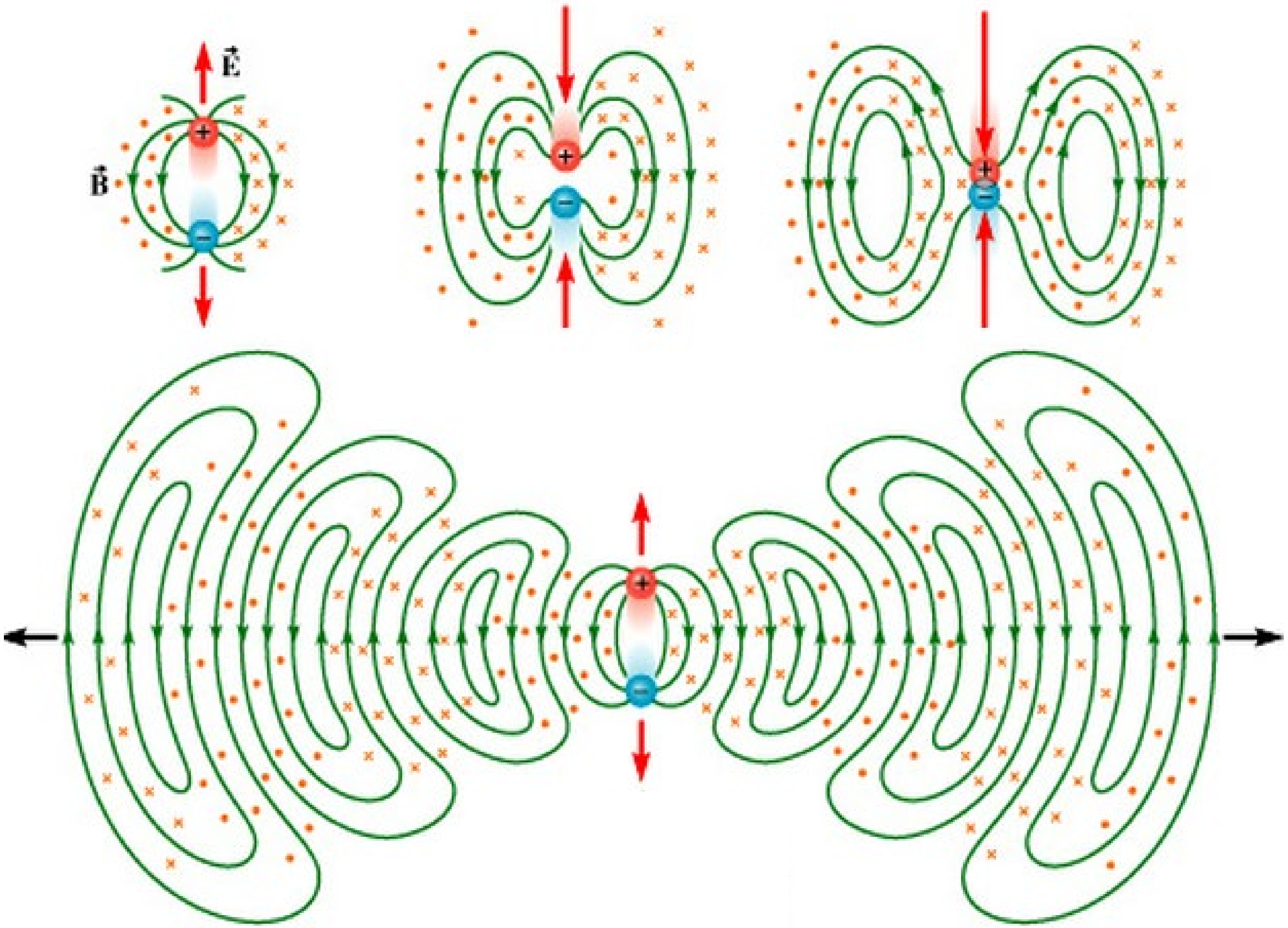


Field created by an oscillating electric dipole



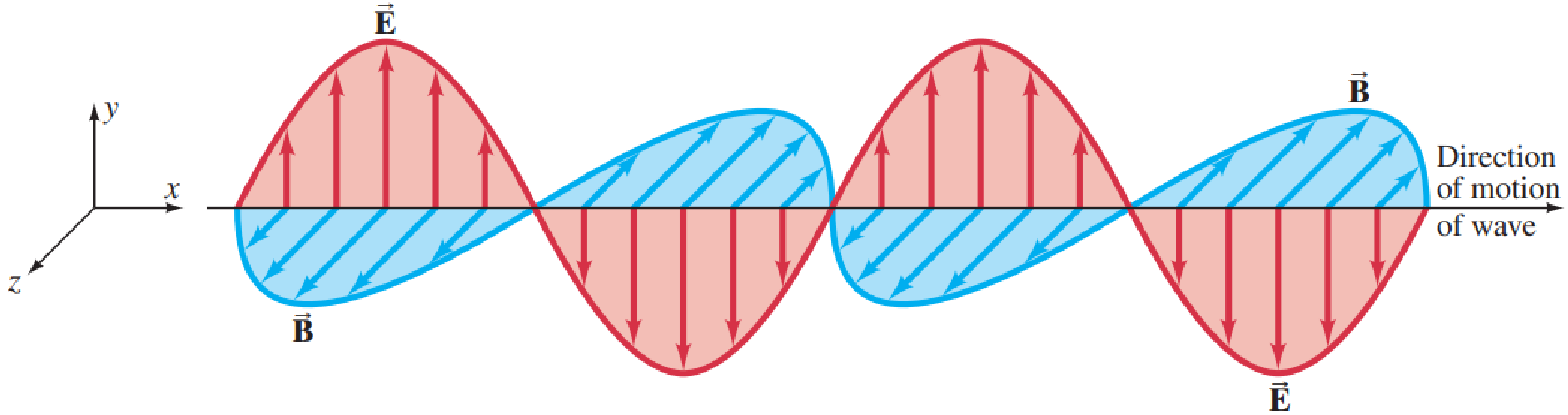
(animation)

https://upload.wikimedia.org/wikipedia/commons/a/a6/Dipole_xmting_antenna_animation_4_408x318x150ms.gif


$\vec{E}$
$\vec{B}$

Production of Electromagnetic Waves

If the source voltage varies sinusoidally, then the electric and magnetic field strengths in the radiation field will also vary sinusoidally. The sinusoidal character of the waves is diagrammed in the figure below, which shows the field directions and magnitudes plotted as a function of position.
Notice that **B** and **E** are perpendicular to each other and to the direction of travel (=the direction of the wave velocity **v**). The direction of **v** can be found from a right-hand rule using **E**x**B**.



- We call these waves electromagnetic (EM) waves. They are transverse waves because the amplitude is perpendicular to the direction of wave travel.
- EM waves are always waves of fields, not of matter (like waves on water or a rope).
- Because they are fields, EM waves can propagate **in empty space**.
- EM waves are produced by electric charges that are oscillating and hence are undergoing acceleration. **Accelerating electric charges give rise to electromagnetic waves.**

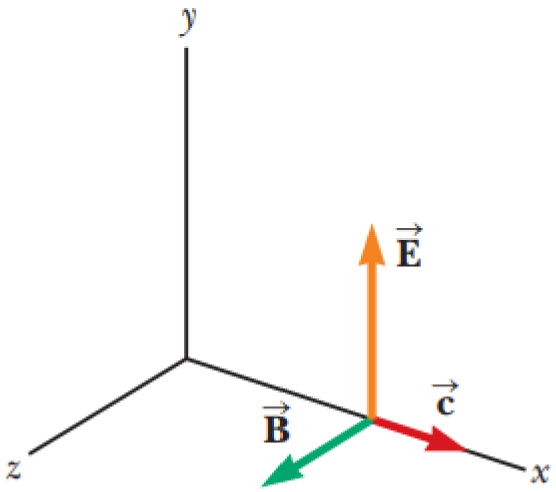
Electromagnetic Waves in vacuum, and Their Speed, Derived from Maxwell's Equations

Let us now examine how the existence of EM waves follows from Maxwell's equations.

We begin by considering **a region of free space, where there are no charges or conduction currents**—that is, far from the source so that the wave fronts are essentially flat over a reasonable area.

We call them plane waves, as we saw, because at any instant **B** and **E** are uniform over a reasonably large plane perpendicular to the direction of propagation.

We choose a coordinate system, so that the wave is traveling in the x direction with velocity **v** with **E** parallel to the y axis and **B** parallel to the z axis.



Maxwell's equations in vacuum, with *Q=I=0 (no sources)*, become:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}.$$

EPFL

Electromagnetic Waves, and Their Speed, Derived from Maxwell's Equations

in vacuum

Maxwell's equations, with *Q=I=0 :*

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}.$$

Notice the **beautiful symmetry** of these equations. The term on the right in the last equation, conceived by Maxwell, is essential for this symmetry. It is also essential if electromagnetic waves are to be produced, as we will now see.

If the wave is sinusoidal with wavelength λ and frequency f, then such a traveling wave can be written as

$$E = E_y = E_0 \sin(kx - \omega t)$$
$$B = B_z = B_0 \sin(kx - \omega t)$$

speed of the wave

$$k = \frac{2\pi}{\lambda}, \qquad \omega = 2\pi f, \qquad \text{and} \qquad f\lambda = \frac{\omega}{k} = v,$$

Such waves, in which the electric and magnetic fields are restricted to being parallel to a pair of perpendicular axes, are said to be linearly polarized waves.

Electromagnetic Waves, and Their Speed, Derived from Maxwell's Equations

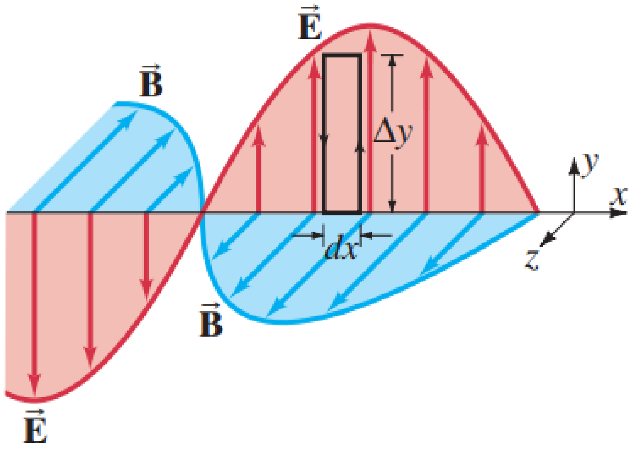


FIGURE 10 Applying Faraday's law to the rectangle $(\Delta y)(dx)$.

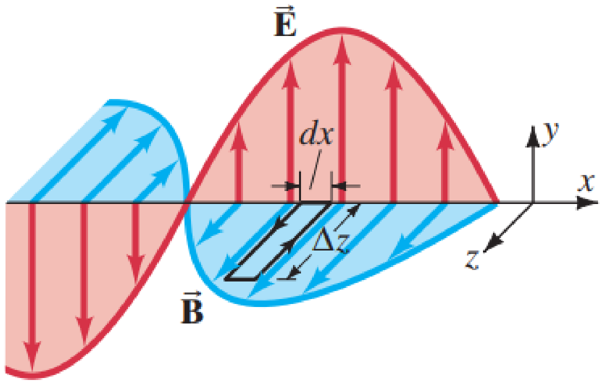


FIGURE 11 Applying Maxwell's fourth equation to the rectangle $(\Delta z)(dx)$.

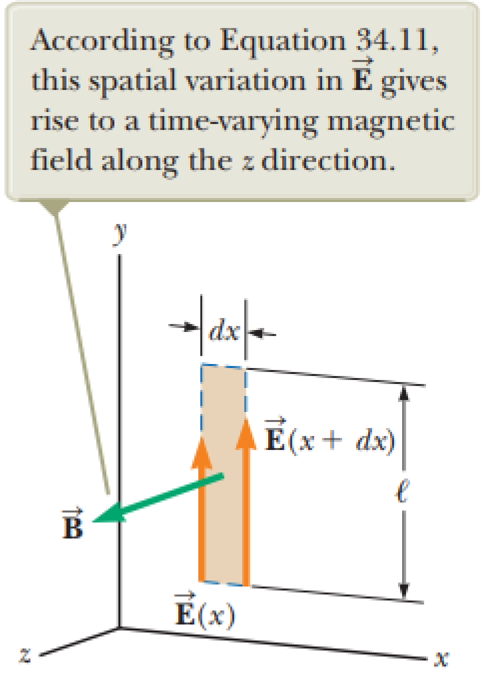


Figure 34.6 At an instant when a plane wave moving in the positive x direction passes through a rectangular path of width dx lying in the xy plane, the electric field in the y direction varies from $\vec{E}(x)$ to $\vec{E}(x + dx)$.

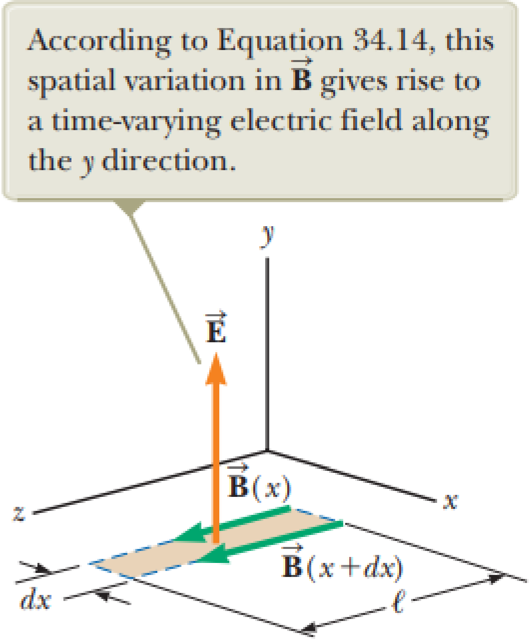


Figure 34.7 At an instant when a plane wave passes through a rectangular path of width dx lying in the xz plane, the magnetic field in the z direction varies from $\vec{B}(x)$ to $\vec{B}(x + dx)$.

Let's consider those circuits and let's apply Faraday's law and Ampere/Maxwell's law.

Electromagnetic Waves, and Their Speed, Derived from Maxwell's Equations

$\Phi_B = B\ell\,dx$ assuming dx is very small compared with the wavelength of the wave

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Faraday's law applied to the rectangle in Figure

Consider a rectangle of width dx and height l, lying in the xy plane as shown in Figure. Let's first evaluate the line integral of $\boldsymbol{E}\cdot d\boldsymbol{s}$ around this rectangle in the counterclockwise direction at an instant of time when the wave is passing through the rectangle.

The contributions from the top and bottom of the rectangle are zero because $\boldsymbol{E}$ is perpendicular to $d\boldsymbol{s}$ for these paths.
We can express the electric field on the right side of the rectangle as

$$E(x+dx) \approx E(x) + \left.\frac{dE}{dx}\right|_{t\text{ constant}} dx = E(x) + \frac{\partial E}{\partial x}dx \Rightarrow$$

$$\Rightarrow \oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = [E(x+dx)]\ell - [E(x)]\ell \approx \ell\left(\frac{\partial E}{\partial x}\right)dx$$

$$\frac{d\Phi_B}{dt} = \ell\,dx\left.\frac{dB}{dt}\right|_{x\text{ constant}} = \ell\,dx\frac{\partial B}{\partial t}$$

$$\ell\left(\frac{\partial E}{\partial x}\right)dx = -\ell\,dx\frac{\partial B}{\partial t}$$

$$\boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}}$$

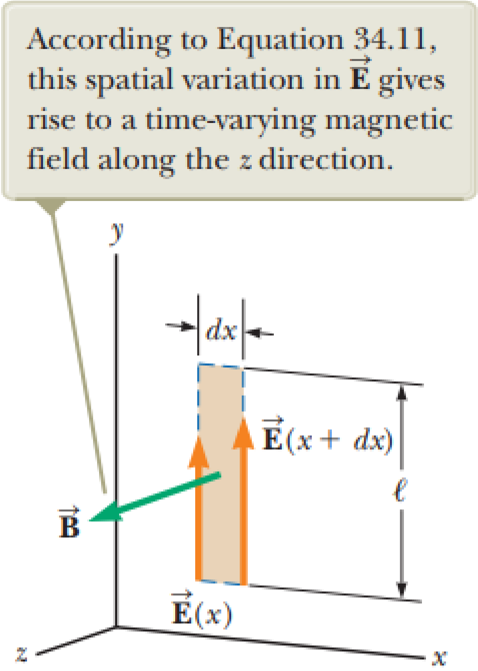


Figure 34.6 At an instant when a plane wave moving in the positive x direction passes through a rectangular path of width dx lying in the xy plane, the electric field in the y direction varies from $\vec{E}(x)$ to $\vec{E}(x+dx)$.

Noting that the magnitude of the magnetic field changes from B(x) to B(x+dx) over the width *dx* and that the direction for taking the line integral is counterclockwise when viewed from above in Figure

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = [B(x)]\ell - [B(x + dx)]\ell \approx -\ell\left(\frac{\partial B}{\partial x}\right) dx$$

$$\Phi_E = E\ell\,dx, \qquad \frac{\partial \Phi_E}{\partial t} = \ell\; dx \frac{\partial E}{\partial t}$$

$$-\ell\left(\frac{\partial B}{\partial x}\right) dx = \mu_0\, \epsilon_0\, \ell\; dx \left(\frac{\partial E}{\partial t}\right)$$

$$\boxed{\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}}$$

Ampère's law

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \cancel{\mu_0 I} + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

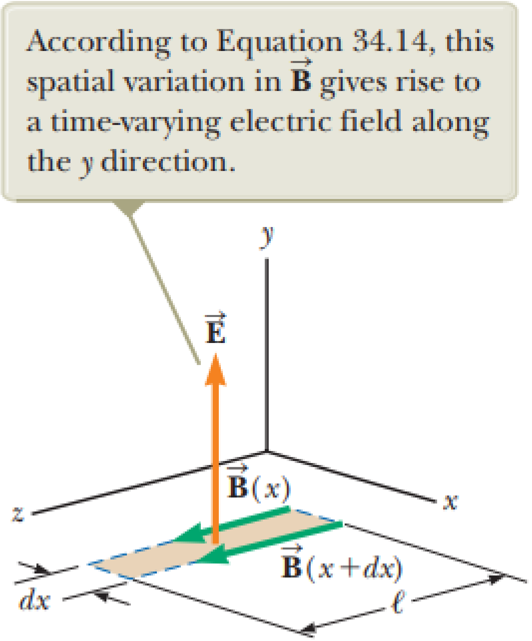


Figure 34.7 At an instant when a plane wave passes through a rectangular path of width *dx* lying in the *xz* plane, the magnetic field in the *z* direction varies from $\vec{\mathbf{B}}(x)$ to $\vec{\mathbf{B}}(x + dx)$.

Slide 13.38 Slide 13.39

Consider $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$ and $\frac{\partial B}{\partial x} = -\mu_0\epsilon_0 \frac{\partial E}{\partial t}$

$$\frac{\partial}{\partial x} \Rightarrow \frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right) = -\frac{\partial}{\partial t}\left(-\mu_0\epsilon_0 \frac{\partial E}{\partial t}\right) \Rightarrow \frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

Remember: (slide 13.6) **Equation of d'Alembert**

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 \psi}{\partial x^2}$$

In a similar way, one can show that it is possible to obtain:

$$\frac{\partial^2 B}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

These equations both have the form of the linear wave equation with the wave speed *v* replaced by **c**, where:

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$c = \frac{1}{\sqrt{(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(8.854\,19 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})}}$$

$$= 2.997\,92 \times 10^{8}\ \mathrm{m/s}$$

Because this speed is precisely the same as the speed of light in empty space, we are led to believe (correctly) that

light is an electromagnetic wave !!!

EPFL

The simplest solution of those wave equations is a sinusoidal wave for which the field magnitudes E and B vary with x and t according to the expressions:

$$(1) \quad E = E_{\max} \cos(kx - \omega t)$$

$$(2) \quad B = B_{\max} \cos(kx - \omega t)$$

where E_{max} and B_{max} are the maximum values of the fields.

The angular wave number is $k = \frac{2\pi}{\lambda}$, where λ is the wavelength.

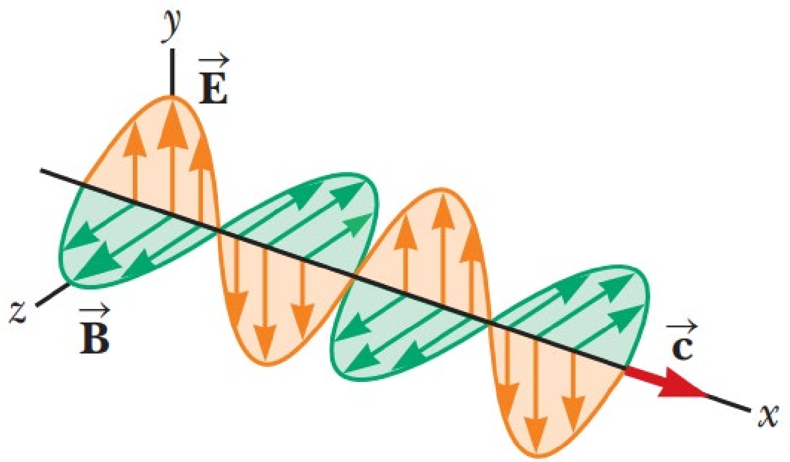
The angular frequency is $\omega = 2\pi f$, where f is the wave frequency. According to the traveling wave model, the ratio ω/k equals the speed of an electromagnetic wave, c

$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f = c$$

where we have used $v = c = \lambda f$, which relates the speed, frequency, and wavelength of a sinusoidal wave.

Therefore, for electromagnetic waves, the wavelength and frequency of these waves are related by

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{f}$$



Pictorial representation, at one instant, of a sinusoidal, linearly polarized electromagnetic wave moving in the positive x direction.

Taking partial derivatives of Equation (1) (with respect to x) and (2) (with respect to t) gives

(1) $E = E_{max} \cos(kx - \omega t)$

(2) $B = B_{max} \cos(kx - \omega t)$

$$\frac{\partial E}{\partial x} = -kE_{max} \sin(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = \omega B_{max} \sin(kx - \omega t)$$

Slide 13.38

place in ⇒ $$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$kE_{max} = \omega B_{max}$$

⇒ $$\frac{E_{max}}{B_{max}} = \frac{\omega}{k} = c$$

$$\frac{E_{max}}{B_{max}} = \frac{E}{B} = c$$

That is, **at every instant**, the ratio of the magnitude of the electric field to the magnitude of the magnetic field in an electromagnetic wave equals the speed of light.

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{1}{\sqrt{(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})}} = 3.00 \times 10^8\ \mathrm{m/s}.$$

This is a remarkable result. For this is precisely equal to the measured speed of light!

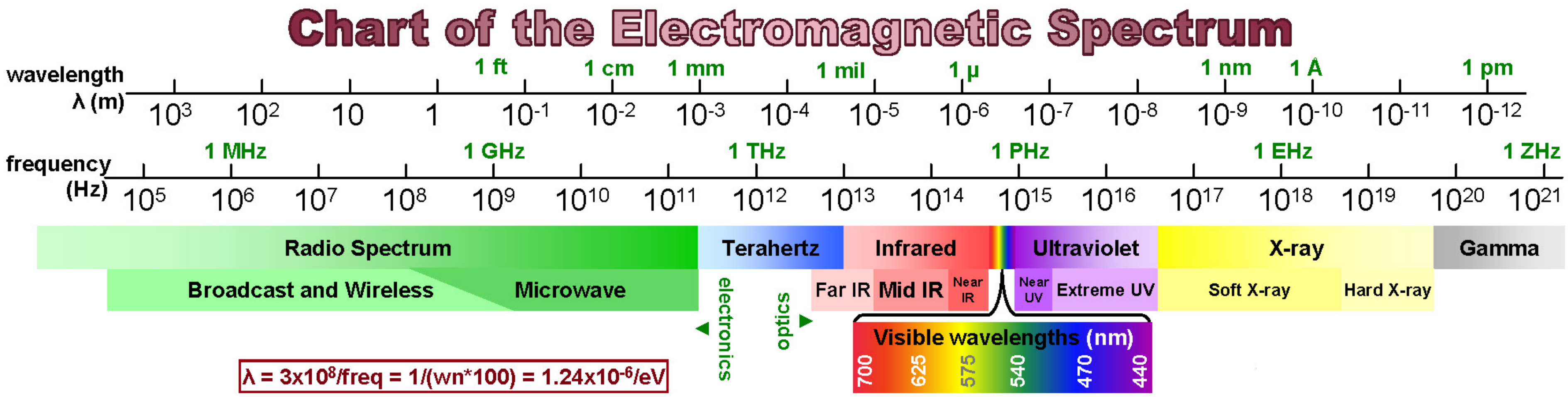
Light as an Electromagnetic Wave and the Electromagnetic Spectrum

$$c = \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m/s}$$

The wavelengths of visible light were measured in the first decade of the nineteenth century, long before anyone imagined that light was an electromagnetic wave. The wavelengths were found to lie between 4.0×10^{-7} m and 7.5×10^{-7} m, or 400 nm to 750 nm $(1 \text{ nm} = 10^{9} \text{ m})$. The frequencies of visible light can be found

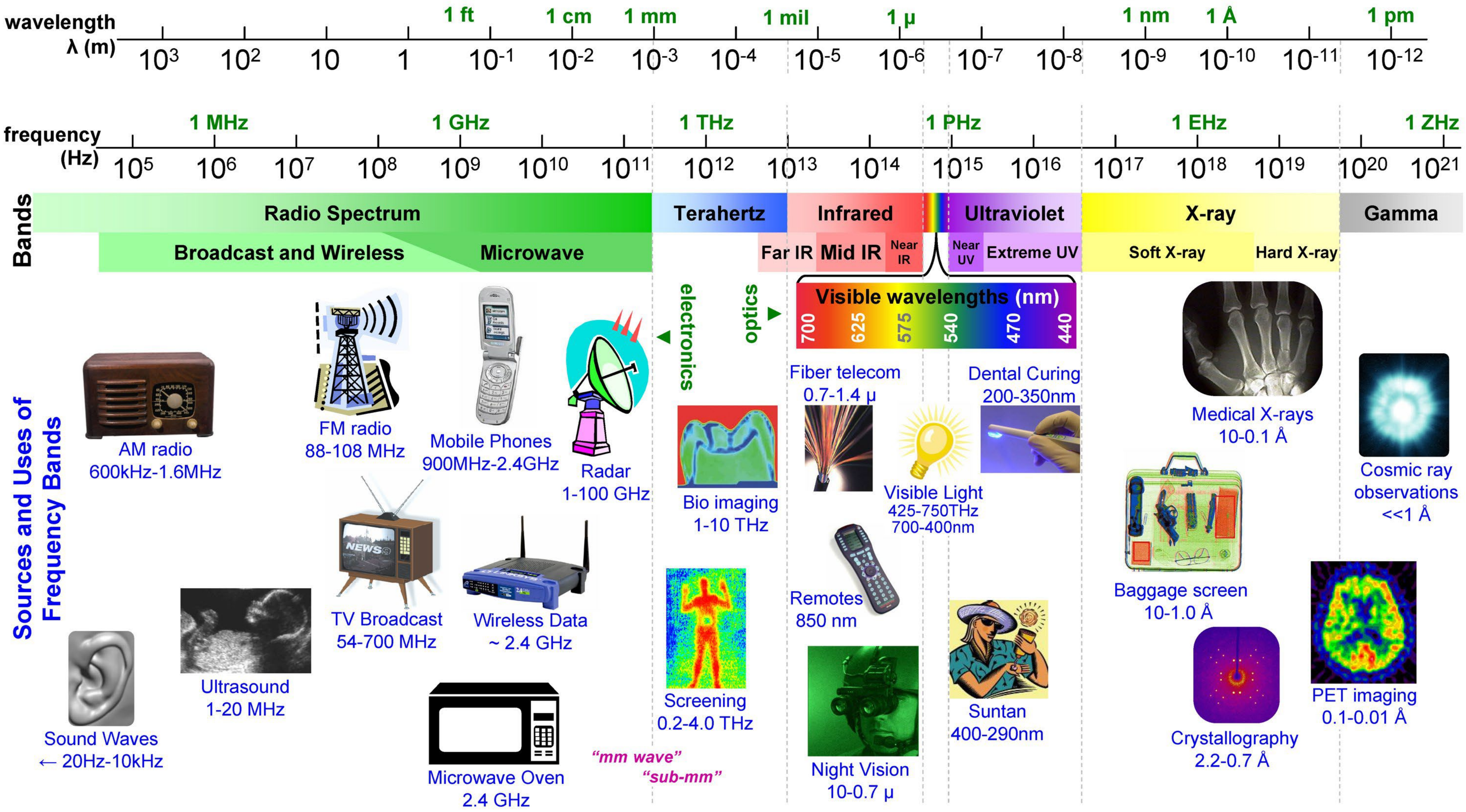
$$c = \lambda f$$

where f and λ are the frequency and wavelength, respectively, of the wave. Here, c is the speed of light, 3.00×10^8 m/s; it gets the special symbol c because of its universality for all EM waves in free space.



- enormous range of wave lengths and frequencies
- spans more than 15 orders of magnitude

Applications of electromagnetic waves



$\lambda = 3\times10^{8}/freq = 1/(wn*100) = 1.24\times10^{-6}/eV$

Energy in EM Waves

Electromagnetic waves carry energy from one region of space to another. This energy is associated with the moving electric and magnetic fields. The energy density u_E $(\mathrm{J/m^3})$ stored in an electric field E is $u_E = \frac{1}{2}\epsilon_0 E^2$. The energy density stored in a magnetic field B is given by $u_B = \frac{1}{2}B^2/\mu_0$. Thus, the total energy stored per unit volume in a region of space where there is an electromagnetic wave is

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}.$$

(total energy density associated to an EM wave)

In this equation, E and B represent the electric and magnetic field strengths of the wave at any instant in a small region of space. We can write u in terms of the E field alone, using $(B = E/c)$ and $(c = 1/\sqrt{\epsilon_0\mu_0})$ to obtain

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{\epsilon_0\mu_0 E^2}{\mu_0} = \epsilon_0 E^2.$$

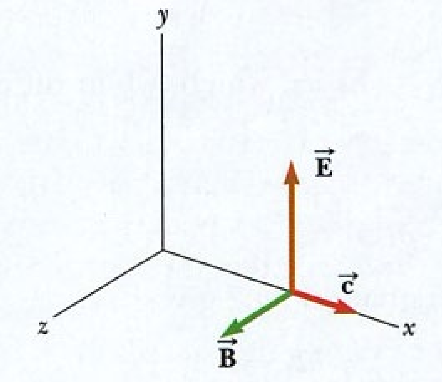
Note here that the energy density associated with the B field equals that due to the E field, and each contributes half to the total energy. We can also write the energy density in terms of the B field only:

$$u = \epsilon_0 E^2 = \epsilon_0 c^2 B^2 = \frac{B^2}{\mu_0},$$

or in one term containing both E and B,

$$u = \epsilon_0 E^2 = \epsilon_0 EcB = \frac{\epsilon_0 EB}{\sqrt{\epsilon_0\mu_0}} = \sqrt{\frac{\epsilon_0}{\mu_0}}\, EB.$$

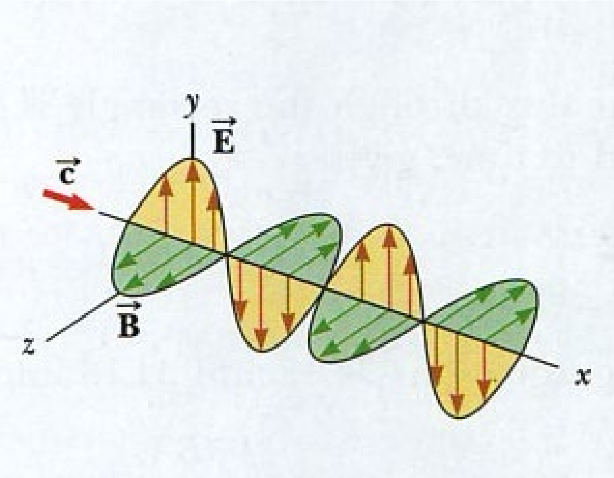
Summary: Electromagnetic plane waves in vacuum



$\mathbf{E}(\mathbf{x},t) = \mathbf{E}_0 \cos(\mathbf{k}\cdot\mathbf{x} - \omega t + \varphi_0)$ $\mathbf{B}(\mathbf{x},t) = \mathbf{B}_0 \cos(\mathbf{k}\cdot\mathbf{x} - \omega t + \varphi_0)$

wave vector: $\mathbf{k}$ pulsation: $\omega = c|\mathbf{k}| = ck$

wavelength: $\lambda = \dfrac{2\pi}{|\mathbf{k}|}$ direction of propagation: $\hat{\mathbf{k}} = \dfrac{\mathbf{k}}{|\mathbf{k}|}$



Link between $\mathbf{E}_0$ et $\mathbf{B}_0$:

$$\mathbf{B}_0 = \frac{k}{\omega}\left(\hat{\mathbf{k}} \times \mathbf{E}_0\right) = \frac{1}{c}\left(\hat{\mathbf{k}} \times \mathbf{E}_0\right) \Rightarrow$$

$\mathbf{E}_0$ and $\mathbf{B}_0$ are:

$-$perpendicular to the direction of propagation" ("transverse wave") (i.e., $\mathbf{B} \perp \mathbf{k}, \mathbf{E} \perp \mathbf{k}$)

$-$perpendicular to each other (i.e., $\mathbf{B} \perp \mathbf{E}$)

$- \ |\mathbf{B}_0| = (1/c)|\mathbf{E}_0|$

Some solved problems

EXAMPLE 2 **Determining $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ in EM waves.** Assume a 60.0-Hz EM wave is a sinusoidal wave propagating in the z direction with $\vec{\mathbf{E}}$ pointing in the x direction, and $E_0 = 2.00\,\text{V/m}$. Write vector expressions for $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ as functions of position and time.

APPROACH We find λ from $\lambda f = v = c$. Then we use Fig. 9 and Eqs. 7 and 8 for the mathematical form of traveling electric and magnetic fields of an EM wave.

SOLUTION The wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\,\text{m/s}}{60.0\,\text{s}^{-1}} = 5.00 \times 10^6\,\text{m}.$$

From Eq. 8 we have

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{5.00 \times 10^6\,\text{m}} = 1.26 \times 10^{-6}\,\text{m}^{-1}$$

$$\omega = 2\pi f = 2\pi(60.0\,\text{Hz}) = 377\,\text{rad/s}.$$

From Eq. 11 with $v = c$, we find that

$$B_0 = \frac{E_0}{c} = \frac{2.00\,\text{V/m}}{3.00 \times 10^8\,\text{m/s}} = 6.67 \times 10^{-9}\,\text{T}.$$

The direction of propagation is that of $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$, as in Fig. 9. With $\vec{\mathbf{E}}$ pointing in the x direction, and the wave propagating in the z direction, $\vec{\mathbf{B}}$ must point in the y direction. Using Eqs. 7 we find:

$$\vec{\mathbf{E}} = \hat{\mathbf{i}}(2.00\,\text{V/m})\sin\left[(1.26 \times 10^{-6}\,\text{m}^{-1})z - (377\,\text{rad/s})t\right]$$

$$\vec{\mathbf{B}} = \hat{\mathbf{j}}(6.67 \times 10^{-9}\,\text{T})\sin\left[(1.26 \times 10^{-6}\,\text{m}^{-1})z - (377\,\text{rad/s})t\right]$$

EXAMPLE 3 **Wavelengths of EM waves.** Calculate the wavelength (*a*) of a 60-Hz EM wave, (*b*) of a 93.3-MHz FM radio wave, and (*c*) of a beam of visible red light from a laser at frequency 4.74×10^{14} Hz.

APPROACH All of these waves are electromagnetic waves, so their speed is $c = 3.00 \times 10^8$ m/s. We solve for λ in Eq. 14: $\lambda = c/f$.

SOLUTION (*a*) $$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{60 \text{ s}^{-1}} = 5.0 \times 10^6 \text{ m},$$

or 5000 km. 60 Hz is the frequency of ac current in the United States, and, as we see here, one wavelength stretches all the way across the continental USA.

(*b*) $$\lambda = \frac{3.00 \times 10^8 \text{ m/s}}{93.3 \times 10^6 \text{ s}^{-1}} = 3.22 \text{ m}.$$

The length of an FM antenna is about half this ($\frac{1}{2}\lambda$), or $1\frac{1}{2}$ m.

(*c*) $$\lambda = \frac{3.00 \times 10^8 \text{ m/s}}{4.74 \times 10^{14} \text{ s}^{-1}} = 6.33 \times 10^{-7} \text{ m } (= 633 \text{ nm}).$$

EXAMPLE 4 **ESTIMATE** **Cell phone antenna.** The antenna of a cell phone is often $\frac{1}{4}$ wavelength long. A particular cell phone has an 8.5-cm-long straight rod for its antenna. Estimate the operating frequency of this phone.

APPROACH The basic equation relating wave speed, wavelength, and frequency is $c = \lambda f$; the wavelength λ equals four times the antenna's length.

SOLUTION The antenna is $\frac{1}{4}\lambda$ long, so $\lambda = 4(8.5 \text{ cm}) = 34 \text{ cm} = 0.34$ m. Then $f = c/\lambda = (3.0 \times 10^8 \text{ m/s})/(0.34 \text{ m}) = 8.8 \times 10^8 \text{ Hz} = 880$ MHz.

NOTE Radio antennas are not always straight conductors. The conductor may be a round loop to save space. See Fig. 21b.

EXAMPLE 5 ESTIMATE **Phone call time lag.** You make a telephone call from New York to a friend in London. Estimate how long it will take the electrical signal generated by your voice to reach London, assuming the signal is (*a*) carried on a telephone cable under the Atlantic Ocean, and (*b*) sent via satellite 36,000 km above the ocean. Would this cause a noticeable delay in either case?

APPROACH The signal is carried on a telephone wire or in the air via satellite. In either case it is an electromagnetic wave. Electronics as well as the wire or cable slow things down, but as a rough estimate we take the speed to be $c = 3.0 \times 10^8\,\mathrm{m/s}$.

SOLUTION The distance from New York to London is about 5000 km.

(*a*) The time delay via the cable is $t = d/c \approx (5 \times 10^6\,\mathrm{m})/(3.0 \times 10^8\,\mathrm{m/s}) = 0.017\,\mathrm{s}$.

(*b*) Via satellite the time would be longer because communications satellites, which are usually geosynchronous, move at a height of 36,000 km. The signal would have to go up to the satellite and back down, or about 72,000 km. The actual distance the signal would travel would be a little more than this as the signal would go up and down on a diagonal. Thus $t = d/c \approx 7.2 \times 10^7\,\mathrm{m}/(3 \times 10^8\,\mathrm{m/s}) = 0.24\,\mathrm{s}$.

NOTE When the signal travels via the underwater cable, there is only a hint of a delay and conversations are fairly normal. When the signal is sent via satellite, the delay *is* noticeable. The length of time between the end of when you speak and your friend receives it and replies, and then you hear the reply, is about a half second beyond the normal time in a conversation. This is enough to be noticeable, and you have to adjust for it so you don't start talking again while your friend's reply is on the way back to you.